

## Theoretical Calculation of Beta Decay Spectrum

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### Abstract

This research is the theoretical investigation of the  $\beta$ -decay spectrum of free neutron decay using Fermi's Golden rule. In the  $\beta$ -decay spectrum with respect to the kinetic energies of the beta particle, end point energy gives the upper limit or maximum energy of  $\beta$ -particle. The maximum beta particle energy,  $E_{\beta}^{\max}$  (or)  $Q_{\beta^-}$  of free neutron decay is 0.782 MeV. According to the calculation, the maximum probability of beta decay is observed for beta particle energy of 0.261 MeV (i.e., one third of the end point energy). The average kinetic energy of  $\beta$ -particle is 0.302 MeV which is nearly equal to the one third of maximum energy of emitted beta particle. In this research, two cases of neutrino mass, zero neutrino mass and finite neutrino mass, are also observed for beta decay.

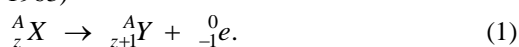
**Keywords:** free neutron decay, maximum beta particle energy.

### Introduction

#### Beta-decay Process

Many unstable nuclides decay by  $\beta$ -decay processes, which are electron emission, positron emission and orbital electron capture, than  $\alpha$ -particle emission. The emission of  $\beta$ -particles differs from that of  $\alpha$ -particles in respect to the spectrum of the energies of the emitted particles. The most characteristic feature of the spontaneous  $\beta$ -disintegration of a nucleus is the continuous distribution in energy of the emitted electrons, which is in sharp contrast to the line-spectra observed for  $\alpha$ -particles.

A nucleus with an overabundance of neutrons can transform to a more stable nucleus by emitting an electron. This kind of process is known as  $\beta$ -decay and the transformation can be denoted by (I. Kaplan, 1963)



where the parent nucleus is at rest, conservation of energy requires

$$E_X = E_Y + E_{e^-}.$$

$$M_X c^2 + 0 = (M_Y c^2 + T_Y) + (T_{e^-} + m_e c^2). \quad (2)$$

The Q-value of the process which is kinetic energy difference between final state and initial state is

$$Q\text{-value} = T_f - T_i$$

$$Q = T_Y + T_{e^-} - 0. \quad (3)$$

From (2) Q-value,  $(T_Y + T_{e^-})$ , is obtained as

$$T_Y + T_{e^-} = (M_X - M_Y - m_e) c^2. \quad (4)$$

Kinetic energy of electron is

$$T_{e^-} = Q - T_Y \approx Q. \quad (5)$$

However, in the  $\beta$ -decay process, total energy and total angular momentum are not conserved because there was an unaccountable loss of energy and angular

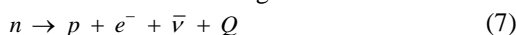
momentum. The observation of  $\beta$ -decay spectra were continuous, which are differing from the line-spectra of  $\alpha$ -decay. In order to explain these problems, in 1931, Pauli introduced the neutrino and antineutrino particles.

#### Neutrino hypothesis

A simple example, the decay of a free neutron, is considered as (W. E. Burchan and M. Jobes, 1979)



The neutron, the proton and the electron have half-integer spin quantum number. Without being the antineutrino on the right-hand side of the above equation, angular momentum cannot be conserved since a possible orbital angular momentum of the electron relative to the proton must have integer quantum number. To fit the requirements of beta emission, the neutrino must be able to carry away energy, momentum, and angular momentum, and must have an inherent spin of  $\frac{1}{2}$ . The spin requirement arises from the fact that in beta decay, single particle of spin  $\frac{1}{2}$  is converted to three particles in accordance with either the following reactions.



where the  $\bar{\nu}$  and  $\nu$  are antiparticle and particle respectively in the same way that  $(e^+ e^-)$  from an antiparticle-particle pair. In either of these reactions, two of the product particles are known to have spins of  $\frac{1}{2}$  each, so their combined spins must be either zero or integral. To maintain spin conservation, the remaining particle, the neutrino, must have a spin of  $\frac{1}{2}$ . According to Pauli, the energy available at each beta emission is divided between beta particle and neutrino in all possible ratios. On the average, the neutrino receives about two times of the energy imparted to the beta. Therefore  $\beta$ -particle receives

about one third of the maximum energy. In 1934, Fermi developed a theory of beta decay, which included the probability of energy division between the two particles.

The kinetic energy of the disintegration products is then,

$$T_p + T_v + T_{\beta^-} = Q - E_x. \quad (9)$$

In beta decay, as in other decay processes, the most important parameters that can be experimentally determined are decay constant and the energies of the emitted radiation.

In this research work, the theoretical calculation of decay probability of the free neutron decay process is observed. The kinematical term of formulation of decay probability can give the continuous spectrum of  $\beta$ -decay. Since the main interest is to observe the continuous  $\beta$ -decay spectrum, only the *kinematical term* (i.e., physical quantities corresponding with movement, for example, momentum and kinetic energy) of formulation of decay probability is considered. The *dynamical term* (i.e., physical quantities corresponding with forces, for example, interaction strength and interaction matrix element) of the formulation is considered as unit of decay probability. In the next section, about the formulation of transition probability will be presented in detail.

### Transition matrix element of beta-decay

The rate of transition of a system from an initial state  $i$  to a final state  $f$  can be expressed as the following form (N. Zettili, 2003)

$$dW_{fi} = \frac{2\pi}{\hbar} |H_{fi}|^2 \rho(E_0). \quad (10)$$

In this formula,

$dW_{fi}$  = decay probability

$|H_{fi}|$  = transition matrix element

$\rho(E_0)$  = number of allowed state or phase-space factor

$E_0$  = the total energy of final state.

The energy  $|H_{fi}|$  is a matrix element connecting the basic interaction of  $\beta$ -decay with wave functions describing the particles concerned in the process.

$$|H_{fi}|^2 = \left| \langle \Psi_f | H_{fi} | \Psi_i \rangle \right|^2. \quad (11)$$

The initial state, with wave function  $\Psi_i$ , is a nucleus containing a number of neutrons, some or all of which are in states from which they are energetically able to transform into a proton. The final state, with wave function  $\Psi_f$ , is a nucleus with one more proton and one less neutron than the initial state, together with an electron and an antineutrino (wave functions  $\Phi_e$  and

$\Phi_{\bar{\nu}_e}$ ). Fermi's assumption of a point like interaction then leads to a decay matrix element,

$$H_{fi} = G \int (\Psi_f^* \Phi_e^* \Phi_{\bar{\nu}_e}^*) \Psi_i d^3r \quad (12)$$

where the integration for  $\Psi_f^* \Psi_i$  is over the nuclear volume and  $G$  represents the strength of the interaction. The integration for the lepton wave function is confined to the assumed final state volume  $V$  within which each is normalized by the factor  $1/\sqrt{V}$ . Taking this normalization into account in Eq.

(12) that because  $|H_{fi}|$  is energy;  $G$  must have the dimensions of the product of energy and volume.

It is clear from the form of the matrix element that the decay probability depends on the overlap between the initial and final wave functions and would be a maximum if the final proton could enter the same state of motion as the initial neutron. The electron and antineutrino are emitted as plane waves

$$\Phi_e \propto \frac{\exp(i\vec{k}_e \cdot \vec{r})}{\sqrt{V}}, \quad \Phi_e^* \propto \frac{\exp(-i\vec{k}_e \cdot \vec{r})}{\sqrt{V}} \quad (13)$$

$$\Phi_{\bar{\nu}_e} \propto \frac{\exp(i\vec{k}_{\bar{\nu}_e} \cdot \vec{r})}{\sqrt{V}}, \quad \Phi_{\bar{\nu}_e}^* \propto \frac{\exp(-i\vec{k}_{\bar{\nu}_e} \cdot \vec{r})}{\sqrt{V}}. \quad (14)$$

So that the lepton part of the matrix element becomes the

$$\begin{aligned} \Phi_e^* \Phi_{\bar{\nu}_e}^* &= \frac{\exp[-i(\vec{k}_e + \vec{k}_{\bar{\nu}_e}) \cdot \vec{r}]}{V} \\ &= \frac{\exp[-i(\vec{k} \cdot \vec{r})]}{V} \end{aligned} \quad (15)$$

where  $\vec{k} = \vec{k}_e + \vec{k}_{\bar{\nu}_e}$  and the  $\vec{k}$ s are de Broglie wave vectors, either for an individual particle or for the relative motion and  $\vec{r}$  is the nuclear radius. The exponential may be expanded as

$$e^{i(\vec{k} \cdot \vec{r})} = 1 + i(\vec{k} \cdot \vec{r}) + \frac{(i(\vec{k} \cdot \vec{r}))^2}{2!} + \dots + \frac{(i(\vec{k} \cdot \vec{r}))^n}{n!}. \quad (16)$$

Because  $\vec{k} \cdot \vec{r}$  is small successive terms of the expansions decrease rapidly and it is now possible to associate  $\ell = 0$  with allowed transitions and  $\ell \geq 1$  with the forbidden transitions of lower probability. If we consider only the allowed transitions we take the first term of the lepton wave function expansion (i.e., unity) and write the remaining part of the matrix elements as

$$\frac{G}{V} \int \Psi_f^* \hat{\tau} \Psi_i d^3r = \frac{G M_{fi}}{V} \quad (17)$$

where  $M_{fi}$  is the nuclear matrix element for the simple point interaction and  $\hat{\tau}$  is an operator that changes a neutron into a proton.

### Transition probability for a constant perturbation

In the case where  $\hat{V}$  does not depend on time, the transition probability for a constant perturbation leads to

$$P_{if}(t) = \left| -\frac{i}{\hbar} \int_0^t \langle \Psi_f | \hat{V} | \Psi_i \rangle e^{i\omega_{fi}t'} dt' \right|^2$$

$$= \left| -\frac{i}{\hbar} \langle \Psi_f | \hat{V} | \Psi_i \rangle \frac{e^{i\omega_{fi}t} - 1}{i\omega_{fi}} \right|^2$$

$$= \frac{1}{\hbar^2 \omega_{fi}^2} \left| \langle \Psi_f | \hat{V} | \Psi_i \rangle \right|^2 \left| e^{i\omega_{fi}t} - 1 \right|^2. \quad (18)$$

$$\text{where } \left| e^{i\omega_{fi}t} - 1 \right|^2 = 4 \sin^2(\omega_{fi}t/2) \quad (19)$$

By substitution the above value into Eq. (18) we get as

$$P_{if}(t) = \frac{4 \left| \langle \Psi_f | \hat{V} | \Psi_i \rangle \right|^2}{\hbar^2 \omega_{fi}^2} \sin^2(\omega_{fi}t/2). \quad (20)$$

As a function of time, this transition probability is an oscillating sinusoidal function with a period of  $2\pi/\omega_{fi}$ . Here, for a fixed  $t$ , it has been assumed that  $\omega_{fi}$  is a continuous variable; that is, it has been considered a continuum of final states. When the shape of sinusoidal term,  $\left( \sin^2\left(\frac{\omega_{fi}t}{2}\right) \right) / \omega_{fi}^2$ , has been plotted with respect to  $\omega_{fi}$  for a fixed value of  $t$ , the spectral shape is observed with the central peak (at  $\omega_{fi} = 0$ ). In the limit  $t \rightarrow \infty$  the transition probability takes the shape of a delta function. The detail discussions of the shape of the transition probability will be presented in result and discussion.

As  $t \rightarrow \infty$  we can use the asymptotic relation,

$$\lim_{t \rightarrow \infty} \frac{\sin^2(yt)}{y^2} = \pi t \delta(y). \quad (21)$$

This relation is compared with the sinusoidal term,  $4 \sin^2(\omega_{fi}t/2) / \omega_{fi}^2$ , from Eq. (20) as

$$\frac{1}{(\frac{1}{2}\omega_{fi})^2} \sin^2\left(\frac{\omega_{fi}t}{2}\right) = \pi t \delta\left(\frac{\omega_{fi}}{2}\right).$$

$$\frac{1}{(\frac{1}{2}\omega_{fi})^2} \sin^2\left(\frac{\omega_{fi}t}{2}\right) = 2\pi t \hbar \delta(E_f - E_i), \quad (22)$$

$$\text{where } \delta(ax) = \frac{1}{a} \delta(x)$$

$$\delta\left(\frac{\omega_{fi}}{2}\right) = 2\delta(\omega_{fi})$$

$$= 2\hbar \delta(E_f - E_i).$$

We can reduce Eq. (20) in the following expression,

$$P_{if}(t) = \frac{2\pi}{\hbar} \left| \langle \Psi_f | \hat{V} | \Psi_i \rangle \right|^2 \delta(E_f - E_i). \quad (23)$$

The transition rate, which is defined as a transition probability per unit time,

$$dW_{fi} = \frac{P_{if}(t)}{t} = \frac{2\pi}{\hbar} \left| \langle \Psi_f | \hat{V} | \Psi_i \rangle \right|^2 \delta(E_f - E_i). \quad (24)$$

The delta term  $\delta(E_f - E_i)$  guarantees the conservation of energy; in the limit  $t \rightarrow \infty$ , the transition rate is no vanishing only between states of equal energy.

### Transition into a continuum of final states

The total transition rate associated with a transition from an initial state  $|\Psi_i\rangle$  into a continuum of final states  $|\Psi_f\rangle$  is calculated. If  $\rho(E_f)$  is the density of final states, the number of allowed state per unit energy interval, the number of final states within the energy interval  $E_f$  and  $E_f + dE_f$  is equal to  $\rho(E_f)dE_f$ . The total transition rate  $W_{if}$  can then be obtain from Eq. (24),

$$W_{if} = \int \frac{P_{if}(t)}{t} \rho(E_f) dE_f \quad (25)$$

$$= \frac{2\pi}{\hbar} \left| \langle \Psi_f | \hat{V} | \Psi_i \rangle \right|^2 \int \rho(E_f) \delta(E_f - E_i) dE_f$$

$$= \frac{2\pi}{\hbar} \left| \langle \Psi_f | \hat{V} | \Psi_i \rangle \right|^2 \rho(E_i) \quad (26)$$

where  $\int \delta(E_f - E_i) dE_f = 1$ . The above relation is called the *Fermi golden rule*, which is the general formulation of the total transition rate of a system from an initial state  $i$  to a final state  $f$ . Then it will be continued to calculate the total transition rate concerned with the  $\beta$ -decay process which will be elaborated in next section.

### Decay probability for emission of beta-particles with zero neutrino mass ( $m_{\bar{\nu}_e} = 0$ )

It is first assumed that neutrino is considered as zero mass. The number of states per unit energy interval for an electron and antineutrino in a volume  $V$  may be written,

$$\rho(E_0) dE_0 = \frac{V}{h^3} d^3p_e \frac{V}{h^3} d^3p_{\bar{\nu}_e} \quad (27)$$

where  $d^3p = 4\pi p^2 dp$ . The recoil nucleus in the  $\beta$ -decay takes up negligible energy but it absorbs momentum and as far as momenta are concerned we regard the  $\beta$  particles and antineutrino as independent particles. Their total energies are connected by the equation

$$E_0 = E_e + E_{\bar{\nu}_e} \quad (28)$$

$$\text{in which, } E_{\nu e}^2 = p_{\nu e}^2 c^2 + m_{\nu e}^2 c^4,$$

$$p_{\bar{\nu}_e} = (E_0 - E_e)/c. \quad (29)$$

In order to study the momentum spectrum, we have to integrate over the antineutrino momenta associated with  $p_e$  but subject to Eq. (28).

Eq. (27) then gives with the assumption  $E_{\bar{\nu}_e} = p_{\bar{\nu}_e} c$ ,

$$\begin{aligned} \rho(E_0) dE_0 &= \frac{V}{h^3} d^3 p_e \frac{V}{h^3} d^3 p_{\bar{\nu}_e} \\ \frac{d}{dE_0} \rho(E_0) dE_0 &= \frac{d}{dE_0} \frac{V}{h^3} d^3 p_e \frac{V}{h^3} d^3 p_{\bar{\nu}_e} \end{aligned} \quad (30)$$

$$\begin{aligned} \int d\rho(E_0) &= \frac{4\pi p_e^2}{h^3} dp_e V \frac{d}{dE_0} \int_0^{p_{\max}} \frac{4\pi p_{\bar{\nu}_e}^2}{h^3} dp_{\bar{\nu}_e} V \\ \rho(E_0) &= \frac{16\pi^2 V^2}{h^6} p_e^2 dp_e \frac{d}{dE_0} \int_0^{(E_0 - E_e)/c} p_{\bar{\nu}_e}^2 dp_{\bar{\nu}_e} \\ &= \frac{16\pi^2 V^2}{h^6} p_e^2 dp_e \frac{d}{dE_0} \left[ \frac{p_{\bar{\nu}_e}^3}{3} \right]_0^{(E_0 - E_e)/c} \\ &= \frac{16\pi^2 V^2}{h^6} p_e^2 dp_e \frac{d}{dE_0} \left[ \frac{(E_0 - E_e)^3 / c^3}{3} \right] \\ &= \frac{16\pi^2 V^2}{h^6 c^3} p_e^2 dp_e \frac{3(E_0 - E_e)^2}{3} \\ &= \frac{16\pi^2 V^2}{h^6 c^3} (E_0 - E_e)^2 p_e^2 dp_e. \end{aligned} \quad (31)$$

From Eq. (10) and Eq. (17), the decay probability for the emission of  $\beta$  particles with momentum between  $p_e$  and  $p_e + dp_e$ ,

$$\begin{aligned} dW_{fi} &= \frac{2\pi}{\hbar} |H_{fi}|^2 \rho(E_0) \\ &= \frac{2\pi}{\hbar} \frac{G^2}{V^2} |M_{fi}|^2 \frac{16\pi^2 V^2}{h^6 c^3} (E_0 - E_e)^2 p_e^2 dp_e \\ &= \frac{2\pi}{\hbar} \frac{G^2}{V^2} |M_{fi}|^2 \frac{16\pi^2 V^2}{(2\pi\hbar)^6 c^3} (E_0 - E_e)^2 p_e^2 dp_e \\ &= \frac{1}{2\pi^3 \hbar^7 c^3} p_e^2 (E_0 - E_e)^2 dp_e G^2 |M_{fi}|^2 \\ \frac{dW_{fi}}{dp_e} &= \frac{1}{2\pi^3 \hbar^7 c^3} p_e^2 (E_0 - E_e)^2 G^2 |M_{fi}|^2 \\ &= \frac{c^4}{2\pi^3 \hbar^7 c^7} (E_0 - E_e)^2 p_e^2 G^2 |M_{fi}|^2. \end{aligned} \quad (32)$$

### Decay probability for emission of beta-particles with finite neutrino mass ( $m_{\bar{\nu}_e} \neq 0$ )

Then it is assumed that neutrino is considered as finite neutrino mass. If it is assumed that  $m_{\bar{\nu}_e} \neq 0$ , the non-relativistic theory for treating the momentum spectrum of the  $\beta$ -particles is used. Therefore, it is obtained

$$\begin{aligned} E_0 &= E_e + E_{\bar{\nu}_e} \\ E_{\bar{\nu}_e} &= \sqrt{p_{\bar{\nu}_e}^2 c^2 + m_{\bar{\nu}_e}^2 c^4}, \\ E_e &= \sqrt{p_e^2 c^2 + m_e^2 c^4}. \end{aligned} \quad (34)$$

For  $p_{\bar{\nu}_e}^{\max}$ ,  $E_e = m_e c^2$

$$E_0 = m_e c^2 + \sqrt{p_{\bar{\nu}_e}^2 c^2 + m_{\bar{\nu}_e}^2 c^4} \quad (36)$$

It is obtained as

$$\begin{aligned} (E_0 - m_e c^2)^2 &= p_{\bar{\nu}_e}^2 c^2 + m_{\bar{\nu}_e}^2 c^4 \\ p_{\bar{\nu}_e}^2 c^2 &= (E_0 - m_e c^2)^2 - m_{\bar{\nu}_e}^2 c^4 \\ p_{\bar{\nu}_e} &= \left( (E_0 - m_e c^2)^2 - m_{\bar{\nu}_e}^2 c^4 \right)^{1/2} / c. \end{aligned} \quad (37)$$

The phase space factor, Eq. (18) becomes

$$\begin{aligned} \int d\rho(E_0) &= \frac{4\pi p_e^2}{h^3} dp_e V \frac{d}{dE_0} \int_0^{p_{\max}} \frac{4\pi p_{\bar{\nu}_e}^2}{h^3} dp_{\bar{\nu}_e} V \\ \rho(E_0) &= \frac{16\pi^2 V^2}{h^6 c^3} p_e^2 \left( (E_0 - m_e c^2)^2 - m_{\bar{\nu}_e}^2 c^4 \right)^{1/2} (E_0 - m_e c^2) dp_e. \end{aligned} \quad (38)$$

From Eq. (10) and Eq. (17) the decay probability for the emission of  $\beta$ -particles with momentum between  $p_e$  and  $p_e + dp_e$ ,

$$\begin{aligned} dW_{fi} &= \frac{2\pi}{\hbar} |H_{fi}|^2 \rho(E_0) \\ &= \frac{2\pi}{\hbar} \frac{G^2}{V^2} |M_{fi}|^2 \frac{16\pi^2 V^2}{h^6 c^3} p_e^2 \\ &\quad \times \left( (E_0 - m_e c^2)^2 - m_{\bar{\nu}_e}^2 c^4 \right)^{1/2} (E_0 - m_e c^2) dp_e \\ &= \frac{G^2 |M_{fi}|^2}{2\pi^3 \hbar^7 c^3} p_e^2 \left( (E_0 - m_e c^2)^2 - m_{\bar{\nu}_e}^2 c^4 \right)^{1/2} (E_0 - m_e c^2) dp_e \\ \frac{dW_{fi}}{dp_e} &= \frac{c^4 G^2 |M_{fi}|^2}{2\pi^3 \hbar^7 c^7} \left( (E_0 - m_e c^2)^2 - m_{\bar{\nu}_e}^2 c^4 \right)^{1/2} \\ &\quad \times (E_0 - m_e c^2) p_e^2 \end{aligned} \quad (39)$$

The equations (33) and (39) are formulations of decay probability per unit momentum of  $\beta$ -particle concerning with zero neutrino mass and finite neutrino mass respectively. These equations are numerically solved to observe the  $\beta$ -decay spectra. We have considered only the kinematical terms of these equations. For this research work, the dynamical terms are expressed as unit of decay probability,  $G^2 |M_{fi}|^2 c^2 \text{ MeV}^{-3} \text{ fm}^{-7}$ .

## Result and Discussion

### The shape of transition probability

In the equation of transition probability, Eq.(20), it can be seen that the transition probability is a function of time and an oscillating sinusoidal function with a period of  $2\pi/\omega_{fi}$ . In order to observe the variation of transition probability with the various value of  $\omega_{fi}$ , it is sketched the shape of transition probability by using the term,  $\sin^2(\omega_{fi} t' / 2) / \omega_{fi}^2$ . It is assumed that  $\omega_{fi}$  is a continuous variable; i.e., final state is considered as a continuum state. Fig. (1) shows the shape of transition probability for a fixed value of time 't'.



Then it is formed that the transition probability of finding the system in a final state  $|\Psi_f\rangle$  of energy  $E_f$  is greatest only when  $E_i \approx E_f$  or when  $\omega_{fi} \approx 0$ .

In the Fig. (2), three spectra of transition probability for three fixed values of time are compared. It is observed that the height and the width of the main peak, centered around  $\omega_{fi} = 0$ , are proportional to  $t^2$  and  $(1/t)$ , respectively, so the area under the curve is proportional to time 't'. Since the most of the area is under the central peak, the transition probability is also proportional to 't'. The central peak becomes narrower and higher as time increases, since the transition probability grows linearly with time. When the time t is increased to a infinite value this central peak takes the shape of delta function as shown in Fig. (3). Therefore the property of a delta function for asymptotic relation is used in the calculation of transition probability.

### Transition probability for $\beta$ -decay process concerning with zero neutrino mass

Since our main interest is to study only the shape of the  $\beta$ -decay spectrum, we have simply considered only the kinematical term from the formulation of the transition probability for  $\beta$ -decay process. This term can give the spectral shape of the  $\beta$ -decay process. We first assumed that neutrino is considered as zero mass, i.e., the maximum momentum value of emitted neutrino ( $p_{\nu_e}^{\max}$ ) and Q-value are not concerned with the rest energy of neutrino.

Then we plotted the two spectra with respect to the various momenta and various kinetic energies of the emitted  $\beta$  particles. Fig. (4) and (5) are momentum distribution and energy distribution curves of  $\beta$ -decay process respectively. From energy distribution curve it is observed that the end point energy of this spectrum gives the maximum kinetic energy of the emitted  $\beta$ -particles (i.e., Q-value of the neutron decay process). It is also observed that the maximum probability is obtained at 0.261 MeV of  $\beta$ -particle. This energy value is the one third of end point energy. The average energy which is calculated by integrating the energy distribution spectrum is also nearly equal to the one third of the maximum energy of the emitted  $\beta$ -particle, 0.302 MeV. In the next section, the about of  $\beta$ -decay spectrum concerning with the mass of neutrino is discussed.

### $\beta$ -decay spectrum concerning with finite neutrino mass

It is also calculated that the  $\beta$ -decay spectrum concerning with the finite neutrino mass, 0.073 MeV/c<sup>2</sup>. This is chosen arbitrarily to a value much

higher than the realistic upper limit of neutrino mass ( $\sim 35$  eV/c<sup>2</sup>) to enhance the effect near the end point of the spectrum. In Fig.(6), the two spectra concerning with zero and finite neutrino mass are compared. It was observed that the spectrum near the upper limit in the case of finite neutrino mass is abruptly approached to zero. In this case, Q-value is less than that for the case of zero neutrino mass by amount of neutrino mass.

## Conclusion

In the study of the theoretical calculations of  $\beta$ -decay spectrum, decay probability can be calculated from Fermi Golden rule. Since there are three emitted particles (i.e., proton, electron and neutrino) in  $\beta$ -decay process, the Q-value for the process is shared by various kinetic energy values of these three particles, i.e., the electron can emit with various kinetic energies. Hence the spontaneous  $\beta$ -disintegration of a nucleus is the continuous distribution in energy of the emitted electrons, which is in sharp contrast to the line-spectra observed for  $\alpha$ -particles. In comparison of two cases of neutrino mass, the case of finite neutrino mass can give the more exact value of end point energy.

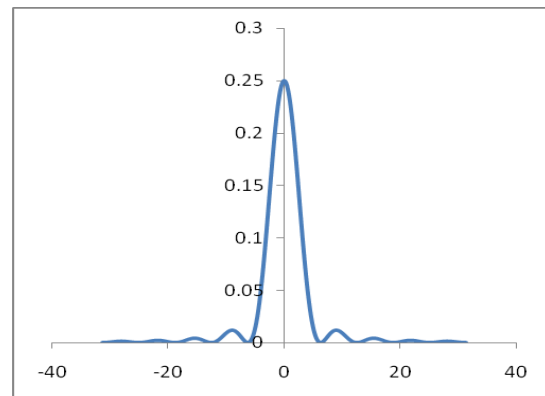


Figure 1.  $\left(\sin^2\left(\frac{\omega_{fi}t}{2}\right)\right)/\omega_{fi}^2$  versus  $\omega_{fi}$  for a fixed value of t;  $\omega_{fi} = ((E_f - E_i)/2)$

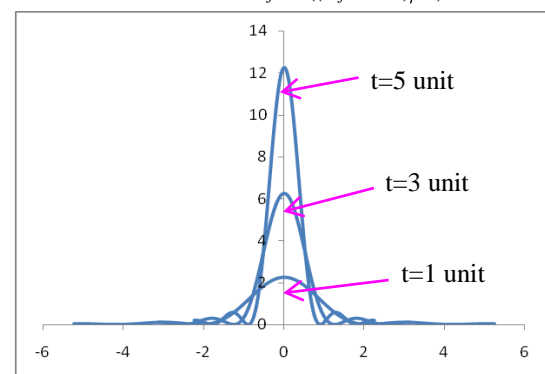


Figure 2. The shape of transition probabilities for (t=1,3,5 unit):

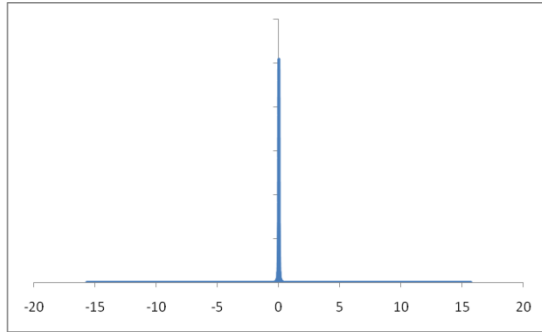


Figure 3. The shape of transition probabilities for  $(t \rightarrow \infty)$ :

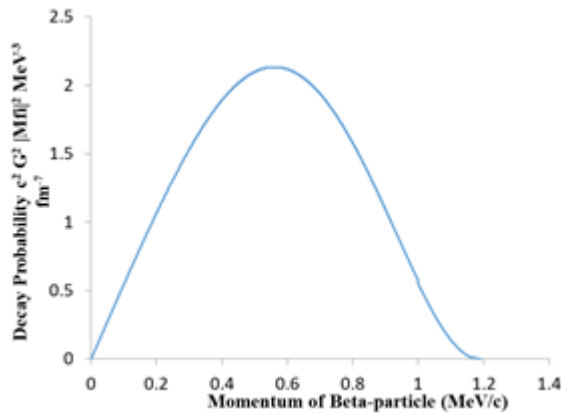


Figure 4.  $\beta$ -decay spectrum with respect to the momentum of  $\beta$  particle.

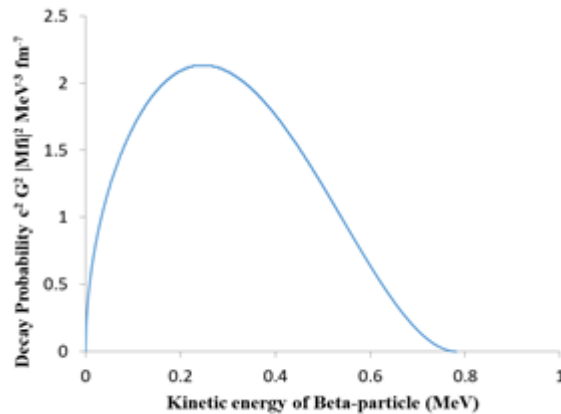


Figure 5.  $\beta$ -decay spectrum with respect to the kinetic energy of  $\beta$  particle

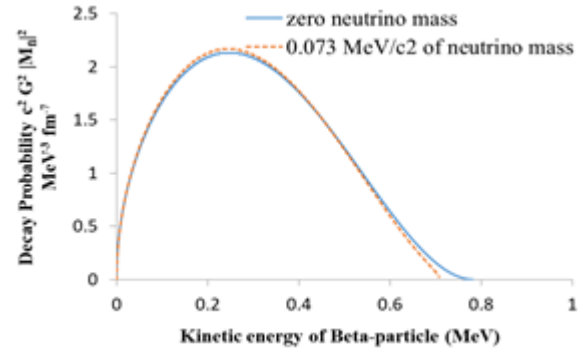


Figure 6. Two energy distribution spectra concerning with zero neutrino mass and finite neutrino mass.

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